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Causality and decoherence in the asymmetric states

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Abstract

Formal quantum causal analysis is applied to the three-qubit states. 'Quantum causality' means asymmetry in the transfer of information. It leads to different entanglement decay of the system at decoherence of different subsystems. The relationship between decoherence (by dissipation, depolarization and dephasing) and causal connections for the different model states has been studied. Generally, this relationship turns out to be rather sophisticated, although some simple relations have been established. So, the dissipated one-particle party tends to be the effect with respect to the rest of the system, while the depolarized one tends to be the cause. In the asymmetric states, the dissipation and depolarization acting along original causality destroy entanglement to a lesser degree than against it. The decohered internal (inside a subsystem) effect destroys entanglement to a lesser degree than the decohered internal cause.

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1. Introduction

The principle of causality conventionally means only a retardation of the effect relative to the cause. However, in simple situations, one can usually distinguish the cause and effect, implying but not measuring the retardation, which bears witness to their asymmetry. From a formal definition of this asymmetry, the method of causal analysis was born (see, e.g., [1]), which was more recently extended to the quantum variables and was applied to the diversified two- and three-qubit states [2–4]. It was found that if a one-qubit party is dissipated, this party turns out to be the effect, while the remaining party is the cause, which is in accordance with intuitive expectation—the flow of free energy or information is directed from a cause to an effect. But in more complicated situations, results of formal causal analysis proved to be nontrivial.

Zyczkowski and Horodecki [5] were the first to put forward the hypothesis on asymmetry in the transfer of quantum information with respect to its direction. They found a surprising property of the quantum–classical system and called it 'anomalous entanglement decay'—bigger fragility to decoherence of the classical subsystem. Their results have been explained in [2, 4] with the results of causal analysis, in the framework of which such an entanglement decay is not anomalous at all. But it is a particular case of the general question: how does the entanglement decay by different means of decoherence depend on the direction and strength of causal connection?

In this paper, we try to solve this question. In previous works on quantum causal analysis [2–4], only one kind of decoherence—dissipation—was considered, because dissipation has a clear classical analogy. Not without reasons, in classical physics the term *irreversible process* is used as a synonym of the dissipative one. The quantum–classical correspondence of depolarization, which classically can be treated as entering the worst, i.e. uniformly distributed noise, is not perfect. Dephasing does not have a classical particle counterpart at all. Therefore in this paper, we consider all three processes of decoherence. These processes are applied to one of the particles of the different asymmetric three-qubit states. The quantitative measure of quantum causality is compared with the degree of entanglement measured by the negativity. As a result, we obtain the relationship of the entanglement fragility to decoherence with the direction and strength of the original and the induced causality.

In section 2, a short review of the quantum causal analysis formalism is presented. In section 3, the model decohered states are described. In section 4, the results of causal analysis of those states are considered. Section 5 is dedicated to a comparison of the causality and degree of entanglement decay. The general results are summarized in section 6.

2. The essence of quantum causal analysis

The basic idea of causal analysis is the formalization of an intuitive understanding of the asymmetry of a cause and an effect, because of which one can usually distinguish them without measuring retardation. Note that in our approach, physical causality is not to be contrasted with mathematical causality, as was suggested recently [6].

Having computed von Neumann entropies S of a whole quantum system and its subsystems A and B (for two-party partitions), and hence their conditional entropies, it is possible to constitute the independence functions i [2, 4]:

$$i_{B|A} = \frac{S(B|A)}{S(B)}, \quad i_{A|B} = \frac{S(A|B)}{S(A)}, \quad -1 \leqslant i \leqslant 1. \tag{1}$$

Next, the measure of causality c_2 called the course of time (following the notation of Kozyrev's pioneering work on causal mechanics [7]) has been derived [2, 4]:

$$c_2(A, B) = k\frac{(1 - i_{A|B})(1 - i_{B|A})}{i_{A|B} - i_{B|A}}. \tag{2}$$

c_2 is the velocity of irreversible information flow. The coefficient $k = \Delta r/\delta t$k, where Δr is the effective distance between A and B and δt is the time of brachistochrone evolution [8]. k is computed from the Hamiltonian, and since it does not qualitatively influence the c_2 behavior [2, 4], in this paper we suppose that $k = 1$.

By definition, *the cause A and the effect B are subsystems for which* $c_2(A, B) > 0$. $c_2(A, B) < 0$ means that B is the cause and A is the effect. The absence of causality corresponds to $|c_2| \to \infty$. Thus the strength of causality is inversely related to $|c_2|$.

Cramer was the first to distinguish the principles of strong and weak causality [9] (we do not mean another recent approach related to weak measurements [10]). Strong causality corresponds to the usual condition of retardation τ of the effect relative to the cause:

$$c_2 > 0 \Rightarrow \tau > 0, \quad c_2 < 0 \Rightarrow \tau < 0, \quad |c_2| \to \infty \Rightarrow \tau \to 0. \tag{3}$$

Without condition (3) we have weak causality. Weak causality corresponds only to nonlocal correlations and implies, in fact, correlation in reverse time, but only related to unknown states (hence the 'telegraph to the past' is impossible). Although it is not very important for the scope of this work, note that weak causality allows the extraction of information from the future without the well-known classical paradoxes. The experimental possibility of detecting such time reversal phenomena was theoretically predicted by Elitzur and Dolev [11] and was indeed proved for intramolecular teleportation [12] and for macroscopic entanglement, e.g. [13]. Note that, in the models considered below, we nowhere use the axiom (3) and reverse time is allowed.

The most striking difference between quantum causality and the classical one is that the former can exist only in the mixed states [2–4]. In other words, finite quantum causality is possible only in the open systems.

Finally, consider the relationship between the course of time c_2 and the marginal asymmetry $\alpha = S(B)/S(A)$. It is readily shown that $|c_2| \to \infty \Rightarrow \alpha \to 1$. The reverse is not true. Of course, α cannot be used as a measure of causality (in particular, it is possible that $\alpha \neq 1$ for a fully uncorrelated case: $i_{B|A} = i_{A|B} = 1$). But for most of the simple states (including all the models considered below) finite c_2 corresponds to finite asymmetry $\alpha \neq 1$, while infinite c_2 corresponds to symmetry $\alpha = 1$.

3. Models

We consider decoherence of four well-known three-qubit states in order of decreasing symmetry.

1. The Greenberg–Horn–Zeilinger (GHZ) state

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle). \tag{4}$$

2. The W state

$$|W\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle). \tag{5}$$

3. The Coffman–Kundu–Wooters (CKW) state [14, 15]

$$|\mathrm{CKW}\rangle = \frac{1}{\sqrt{2}}|100\rangle + \frac{1}{2}(|001\rangle + |010\rangle). \tag{6}$$

4. The WRr state [16, 17]

$$|\mathrm{WRr}\rangle = \frac{1}{\sqrt{6}}(|001\rangle + |010\rangle - 2|100\rangle). \tag{7}$$

The first qubit of every state we call subsystem A and the second and third ones, subsystems B and C. Any two-party partitions of (4) and (5) are equivalent. In states (6) and (7), the party A sets off from B and C; therefore only parties B, C and AB, AC are equivalent. Since all the states (4)–(7) are pure any of their two–one partitions AB–C, A–BC, etc are causeless ($|c_2| \to \infty$) [2, 4].

Finite causality is potentially possible only in the mixed subsystems A–B, A–C and B–C. But due to the symmetry there is a finite causality only in the states (6) and (7); namely, the computations of [2, 4] have yielded for the state (6): $c_2(A, B) = c_2(A, C) \approx 5.30$ and for the state (7): $c_2(A, B) = c_2(A, C) \approx 3.43$. Thus A is the cause with respect to B and C, and in the WRr state (7) these causal connections are expressed more strongly than in the CKW state (6).

The three kinds of decoherence (of $0 \leqslant p \leqslant 1$ degree) reduce to the following transformations [18, 19]:

1. Dissipation

$$\begin{aligned}
&|0\rangle\langle 0| \to |0\rangle\langle 0|, \\
&|1\rangle\langle 1| \to (1-p)|1\rangle\langle 1| + p|0\rangle\langle 0|, \\
&|1\rangle\langle 0| \to \sqrt{1-p}\,|1\rangle\langle 0|, \\
&|0\rangle\langle 1| \to \sqrt{1-p}\,|0\rangle\langle 1|.
\end{aligned} \tag{8}$$

2. Depolarization

$$
\begin{aligned}
&|0\rangle\langle 0| \to (1-p)\,|0\rangle\langle 0| + p\tfrac{I}{2},\\
&|1\rangle\langle 1| \to (1-p)\,|1\rangle\langle 1| + p\tfrac{I}{2},\\
&|1\rangle\langle 0| \to (1-p)\,|1\rangle\langle 0|,\\
&|0\rangle\langle 1| \to (1-p)\,|0\rangle\langle 1|.
\end{aligned} \tag{9}
$$

3. Dephasing

$$
\begin{aligned}
&|1\rangle\langle 0| \to (1-p)\,|1\rangle\langle 0|,\\
&|0\rangle\langle 1| \to (1-p)\,|0\rangle\langle 1|.
\end{aligned} \tag{10}
$$

We apply (8)–(10) to one of the qubits of (4)–(7). Due to the symmetry of these states it is enough to apply a transformation to any of the qubits of (4) and (5) and we select this qubit to be C. For the states (6) and (7) distinguishable results are achieved by the application of a transformation only to qubits C and A.

We apply (8)–(10) to one of the qubits of (4)–(7). For the states (6) and (7) distinguishable results are achieved by the application of a transformation only to qubits C and A (the transformations of B and C are equivalent).

The resulting mixed state formulae are long. On the other hand, qualitatively, the consequences of asymmetric decoherence proved to be similar for every three-qubit state. But it turns out that the greater the original state asymmetry, the more strongly pronounced these consequences are. Hence, we will consider in detail the decoherence in the most asymmetric WRr state (7) and mention briefly the peculiarities of the other states (4)–(6).

Decoherence of WRr (7) by dissipation (diss), depolarization (depol) and dephasing (deph) of the qubits C and A leads to the following mixed states:

$$
\begin{aligned}
\rho^{\mathrm{diss}\,C} = \frac{1}{3}\Bigg[&\frac{1}{2}\,|010\rangle\langle 010| - |010\rangle\langle 100| - |100\rangle\langle 010|\\
&+2\,|100\rangle\langle 100| + \frac{1-p}{2}\,|001\rangle\langle 001|\\
&+\frac{1}{2}p\,|000\rangle\langle 000| + \frac{\sqrt{1-p}}{2}\,(|001\rangle\langle 010|\\
&-2\,|001\rangle\langle 100| + |010\rangle\langle 001| - 2\,|100\rangle\langle 001|)\Bigg],
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
\rho^{\mathrm{depol}\,C} = \frac{1}{3}\Bigg\{&(1-p)\left[\frac{1}{2}\,(|001\rangle\langle 010| + |010\rangle\langle 001|)\right.\\
&\left.-\,|001\rangle\langle 100| - |100\rangle\langle 001|\right]\\
&+\left(1-\frac{p}{2}\right)\left[\frac{1}{2}\,(|001\rangle\langle 001| + |010\rangle\langle 010|)\right.\\
&\left.-\,|010\rangle\langle 100| - |100\rangle\langle 010| + 2\,|100\rangle\langle 100|\right]\\
&+p\left[\frac{1}{4}\,(|000\rangle\langle 000| + |011\rangle\langle 011|)\right.\\
&\left.-\frac{1}{2}\,(|011\rangle\langle 101| + |101\rangle\langle 011|) + |101\rangle\langle 101|\right]\Bigg\},
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
\rho^{\mathrm{deph}\,C} = \tfrac{1}{6}\,\{&[|001\rangle\langle 001| + |010\rangle\langle 010| - 2\,(|010\rangle\langle 100|\\
&+|100\rangle\langle 010|) + 4\,|100\rangle\langle 100|]\\
&+(1-p)\,[|001\rangle\langle 010| + |010\rangle\langle 001|\\
&-2\,(|001\rangle\langle 100| + |100\rangle\langle 001|)]\},
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
\rho^{\mathrm{diss}\,A} = \tfrac{1}{3}\,\big[&\tfrac{1}{2}\,(|001\rangle\langle 001| + |001\rangle\langle 010| + |010\rangle\langle 001|\\
&+|010\rangle\langle 010|) + 2\,(1-p)\,|100\rangle\langle 100|\\
&+2p\,|000\rangle\langle 000| - \sqrt{1-p}\,(|001\rangle\langle 100|\\
&+|010\rangle\langle 100| + |100\rangle\langle 001| + |100\rangle\langle 010|)\big],
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
\rho^{\mathrm{depol}\,A} = \frac{1}{3}\Bigg\{&(p-1)\,(|001\rangle\langle 100| + |010\rangle\langle 100|\\
&+|100\rangle\langle 001| + |100\rangle\langle 010|)\\
&+\left(1-\frac{p}{2}\right)\left[\frac{1}{2}\,(|001\rangle\langle 001| + |001\rangle\langle 010|\right.\\
&\left.+|010\rangle\langle 001| + |010\rangle\langle 010|) + 2\,|100\rangle\langle 100|\right]\\
&+p\left[|000\rangle\langle 000| + \frac{1}{4}\,(|101\rangle\langle 101| + |101\rangle\langle 110|\right.\\
&\left.+|110\rangle\langle 101| + |110\rangle\langle 110|)\right]\Bigg\},
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
\rho^{\mathrm{deph}\,A} = \frac{1}{3}\Bigg[&\frac{1}{2}\,(|001\rangle\langle 001| + |001\rangle\langle 010|\\
&+|010\rangle\langle 001| + |010\rangle\langle 010| + 4\,|100\rangle\langle 100|)\\
&-(1-p)\,(|001\rangle\langle 100| + |010\rangle\langle 100|\\
&+|100\rangle\langle 001| + |100\rangle\langle 010|)\Bigg].
\end{aligned} \tag{16}
$$

From equations (11)–(16), we computed all the marginal and conditional entropies, then we computed the independent functions i like (1) and, finally, we computed the course of time c_2 like (2) for all the distinguishable two-party partitions. For the same partitions the negativity N as a measure of entanglement has been computed.

4. Causal connections at different kinds of decoherence

The simplest cases of causality induced by dissipation in the originally symmetric GHZ and W states have been considered previously in [3], where it was found that the dissipated party corresponds to the effect (informational sink) that is quite natural. The decohered WRr state, having originally the two causal connections $A \to B$ and $A \to C$, produces a much more rich induced causality distribution. At the beginning, consider an original effect C decoherence (figure 1).

The only pair B–C is originally symmetric and therefore one should expect the same behavior of c_2 as in the W state [3], and indeed in figure 1(a) we observe that the dissipated party tends to be the effect. It is seen that depolarization and dissipation, acting on a one-qubit party, induce opposite directions of causal connection with another party—the depolarized party tends to be the cause, while dephasing does not induce causality. In the pair A–C (figure 1(b)) the same processes lead to another picture. Dissipation

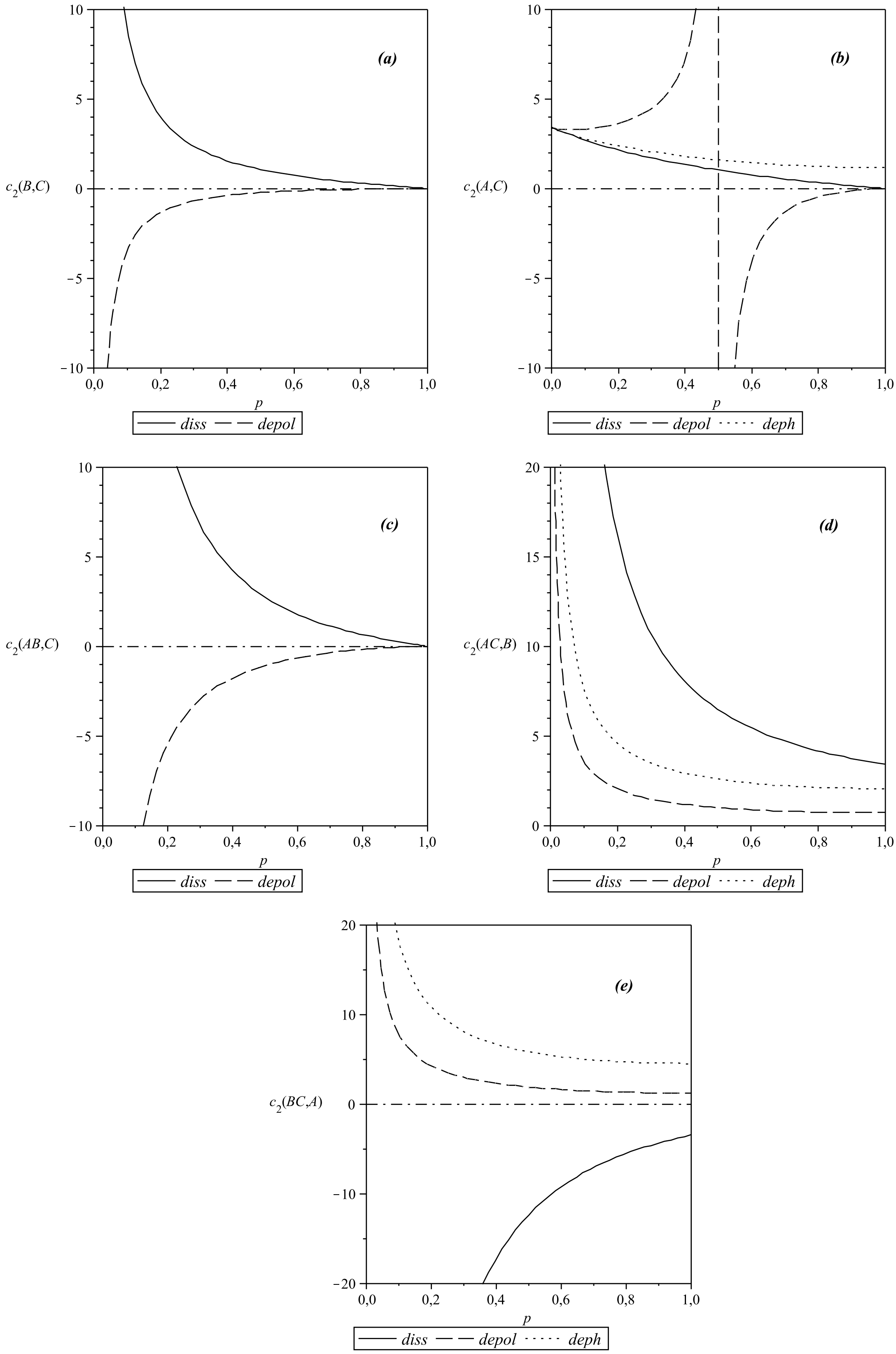

Figure 1. Causality in the WRr state with the decohered qubit C.

and dephasing of C amplify the original causality $A \to C$, while depolarization of C changes the sign of c_2 at $p = \frac{1}{2}$: at $p < \frac{1}{2}$ causality preserves its original direction $A \to C$, and at $p > \frac{1}{2}$ the direction is reversed. The strongest causality is observed in the intuitively expected case of dissipation.

In the case of the partition AB–C (figure 1(c)), we have the same as for the W state [3] and the intuitively expected result: the dissipated party C is the effect with respect to AB, while the depolarized C is the cause. Dephasing does not induce causality.

If the decohered qubit C is included in the two-qubit party AC (figure 1(d)), we observe causality $AC \to B$ at any kind of decoherence. The variation from the W state [3] reduces to the stronger and monotonically amplifying causality for dissipation. The case of the partition A–BC (figure 1(e)) is close, but for depolarization and dephasing $BC \to A$, while for dissipation $A \to BC$. This peculiarity of dissipation is clear. Indeed, at full dissipation ($p = 1$) the particle C 'disappears' from its two-particle party and as a result $c_2(AC, B) = c_2(A, BC) = 3.43$, which is equal to $c_2(A, B)$ at $p = 0$.

The original cause A decoherence leads to a different causal picture (figure 2). One may expect that as a result of the increasing dissipation of A, the original causal connection $A \to C$ will at the beginning attenuate until disappearance at some p; after that the direction of causality will reverse with further utmost amplification of the connection $C \to A$ as p will tend to 1. In figure 2(a) it is seen that indeed $c_2(A, C)$ changes its sign at $p = \frac{3}{4}$ (figure 2(a)). But the variation of positive $c_2(A, C)$ (corresponding to directionality of the causal connection $A \to C$) proves to be not monotonic; it has an intuitively unexpected minimum equal to 2.12 at $p = 0.377$. Next, in the pair A–C (figure 2(a)), depolarization leads to a considerable and monotonic amplification of causality as compared to depolarization of C (figure 1(b)). On the one hand, it is in agreement with intuition (the depolarized A becomes the more intensive information source). On the other hand, it can easily be shown that $S(A)$ and $S(C)$ remain independent of p, which demonstrates that one should not consider the marginal asymmetry α as a sufficient condition or measure of causality.

In the partition AB–C (figure 2(b)) the directionality of the causal connection is $AB \to C$ for any kind of decoherence; therewith the c_2 curves for depolarization and dephasing are monotonic like figure 1(c), while for dissipation the curve has $\min c_2(AB, C) = 1.97$ at $p = 0.627$. The reason why this curve tends to infinity at $p \to 1$ is that at full dissipation the partition AB–C becomes equivalent to the symmetric B–C. And there is an interesting relation, which is valid not only in this model:

$$p\,(\min c_2(AB, C)) = 1 - p\,(|c_2(A, C)| = \infty) + p\,(\min c_2(A, C))\,.$$

The $c_2(A, BC)$ changes its sign at $p = \frac{1}{2}$ (figure 2(c)). At lower values of p, the direction of causal connection is $A \to BC$, and at higher values of p, it is $A \leftarrow BC$. The maximum of $c_2(BC, A) = -15.2$ corresponding to $A \to BC$ is observed at $p = 0.288$. And again the depolarized party A tends to be the cause, while dephasing of this single-particle party does not induce causality.

Now we briefly describe the causal peculiarities of the other states (4)–(6) in order of increasing symmetry.

The CKW state (6) quantitatively looks like the WRr state (7), but has qualitative distinctions under decoherence. Under depolarization of C intuitively we could expect reversal (like that in figure 1(b)) or, at least, attenuation of the original causality, but it turns out amplified, although not monotonically with $\min c_2$ at $p = 0.427$. The reason is that for the CKW state $S(A) = 1 = \max$ and it is impossible to reverse the causal connection without decreasing $S(A)$ below this maximum. The depolarization of C at relatively small p opens the subsystem AC further and amplifies the original causality. At $p \to 1$, $S(C)$ increases up to $S(C) = 1 = \max$

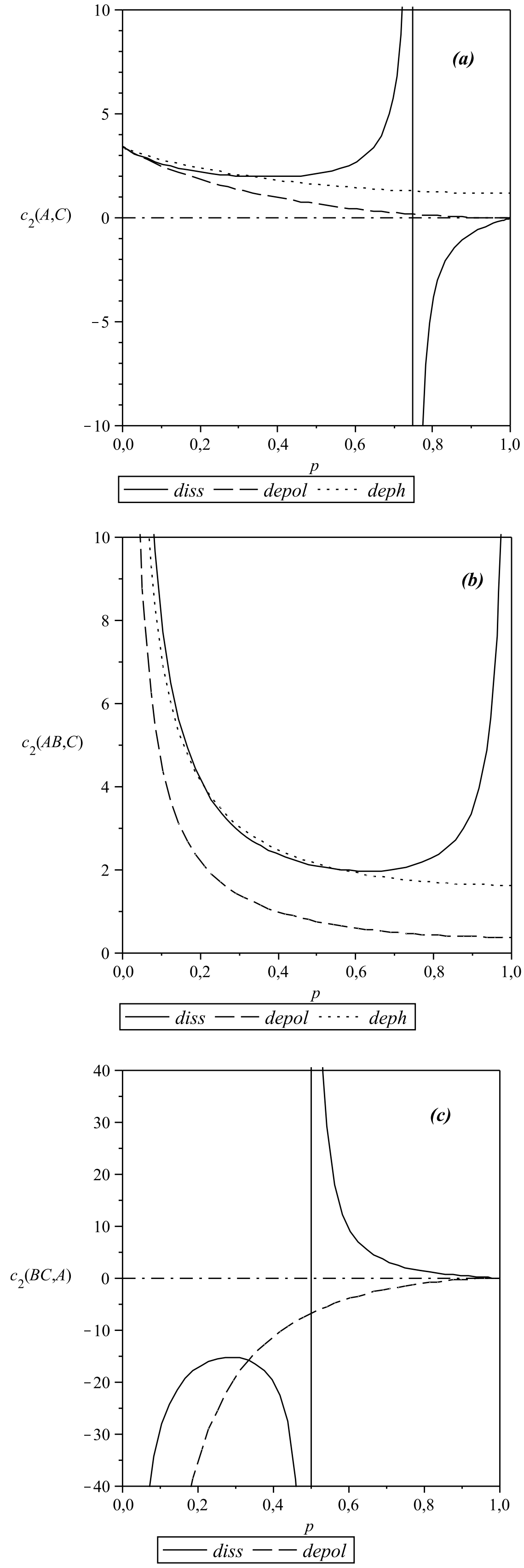

Figure 2. Causality in the WRr state with the decohered qubit A.

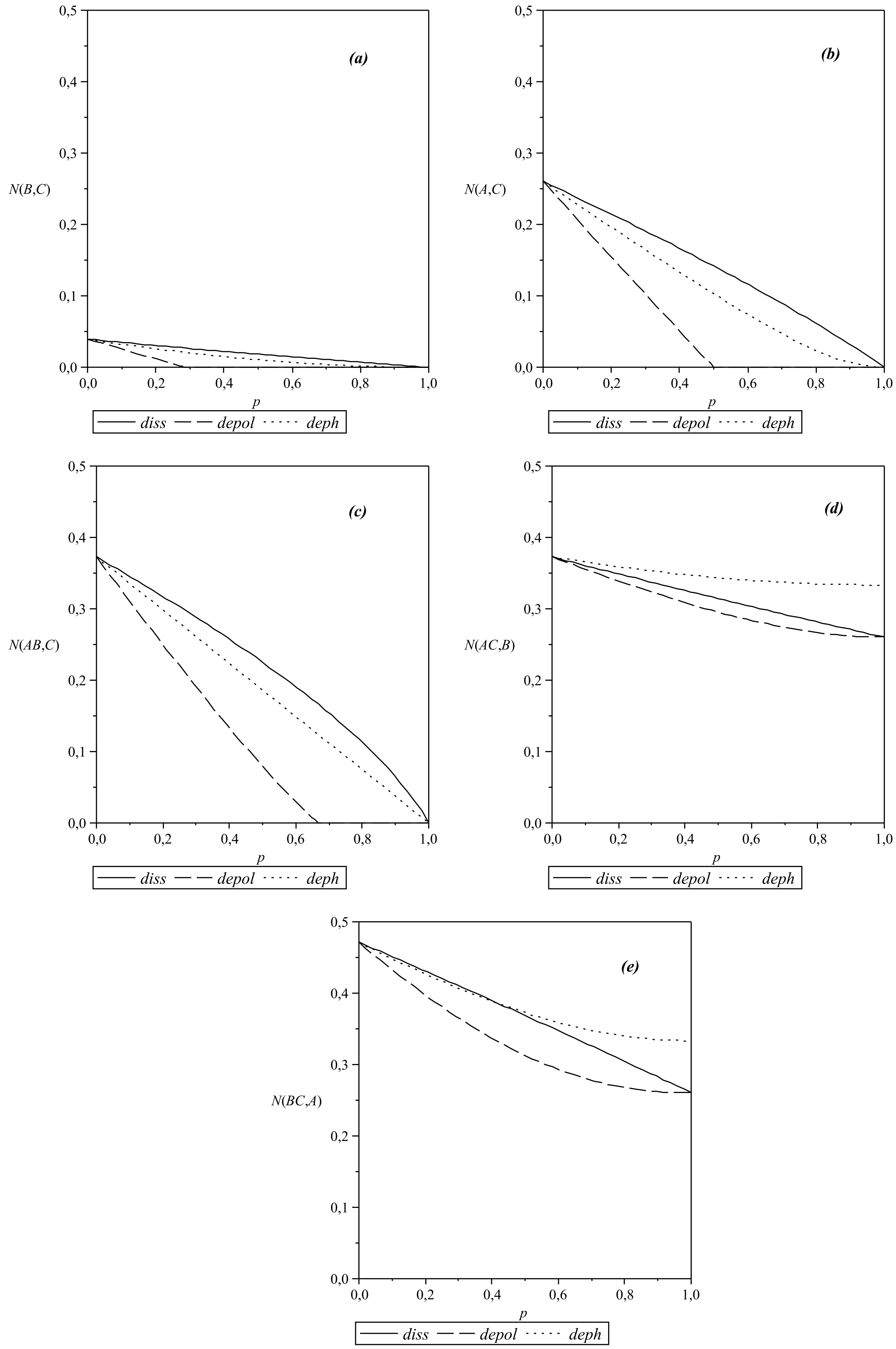

Figure 3. Negativity of the WRr state with the decohered qubit C.

and causality returns to its original level. In contrast, in WRr $S(A) < 1$ and the necessary inequality of the marginal entropies for $C \rightarrow A$ can be achieved.

Under decoherence of A the causality set of CKW still differs from that of WRr. For dissipation of A, $c_2(A, C)$ changes sign at $p = \frac{1}{2}$; that is, the original pairwise causality in the CKW state is less robust than that in the WRr state. The $\min c_2(A, C) = 5.08$ is weaker and is observed now at a lower p, $p = 0.103$. The curves of $c_2(AB, C)$ for the CKW state look like figure 2(b); for dissipation the curve has a minimum at $p = 0.603$. Thus the relation $p\,(\min c_2(AB, C)) = 1 - p\,(|c_2(A, C)| = \infty) + p\,(\min c_2(A, C))$ is valid for the

CKW state as well. In contrast to the WRr state in the partition BC–A (figure 2(c)), in the CKW state in the same partition, depolarization of A does not induce causality at all; only dissipation of A induces the monotonic causal connection; therewith A becomes the effect. The monotonic increase of $c_2(BC, A)$ simply reflects an amplification of causality along with an increase of dissipation of the effect A.

In the decohered W state the causal set is much poorer, described only by three pictures: figures 1(a), (c) and 2(b). Under decoherence for any one of the particles of the W state, the corresponding figures are rather similar but have some quantitative differences.

In the decohered GHZ state the causal set is the poorest. It is also described only by three pictures: figures 1(a), (c) and 2(b). But depolarization induces causality only in the counterpart of figure 2(b) (that is when a particle into the two-particle party is depolarized). Dephasing does not induce causality in any partition.

Summarizing all the results, we can conclude that for dissipation of a one-qubit party this party always becomes an effect. For depolarization and dephasing of a one-qubit party, this party may become only a cause (except A–C in CKW^C, where $S(A) = 1 = \max$). The directionality of causality for dephasing (if it exists) always coincides with that for depolarization (at least for small p in the case of directionality reversal at depolarization). Therewith almost always the dephasing induces a weaker causality than the depolarization.

5. The relation between causality and entanglement decay

All the considered states in any partition (except the pairwise one in the GHZ state) are entangled. Compare the decrease of negativity N of the decohered WRr states (11)–(16) with increasing p, presented in figures 3 and 4, with the c_2 variation in the corresponding figures 1 and 2. It is reasonable to conduct the comparison of N and c_2 at fixed state and kind of decoherence. Therefore, we should consider the original cause (A) decoherence and the original effect (C) decoherence. That is, again we concentrate on the most asymmetric WRr state and mention briefly the peculiarities of the other states.

We begin with the reduced states. Therewith the case of dephasing is irrelevant, because naturally, $N^{\text{deph}\,C} = N^{\text{deph}\,A}$.

In the dissipated WRr states (figures 3(b) and 4(a)), $N^{\text{diss}\,C} < N^{\text{diss}\,A}$. At noted in section 4, dissipation of the original cause A leads to reversal of the original causality (figure 2(a)); dissipation of the original effect C amplifies the original causality (figures 1(b)). As a result $|c_2\,(A, \text{diss}\,C)| < |c_2\,(\text{diss}\,A, C)|$. We conclude that dissipation, amplifying the original causality, destroys entanglement to a lesser extent than dissipation, acting against it.

In the depolarized WRr states (figures 3(b) and 4(a)), $N^{\text{depol}\,C} < N^{\text{depol}\,A}$. As also noted in section 4, the depolarization of the original cause A leads to strong amplification of the original causality (figure 2(a)); the depolarization of the original effect C reverses it (the break in figure 1(b)). As a result, $|c_2\,(A, \text{depol}\,C)| > |c_2\,(\text{depol}\,A, C)|$.

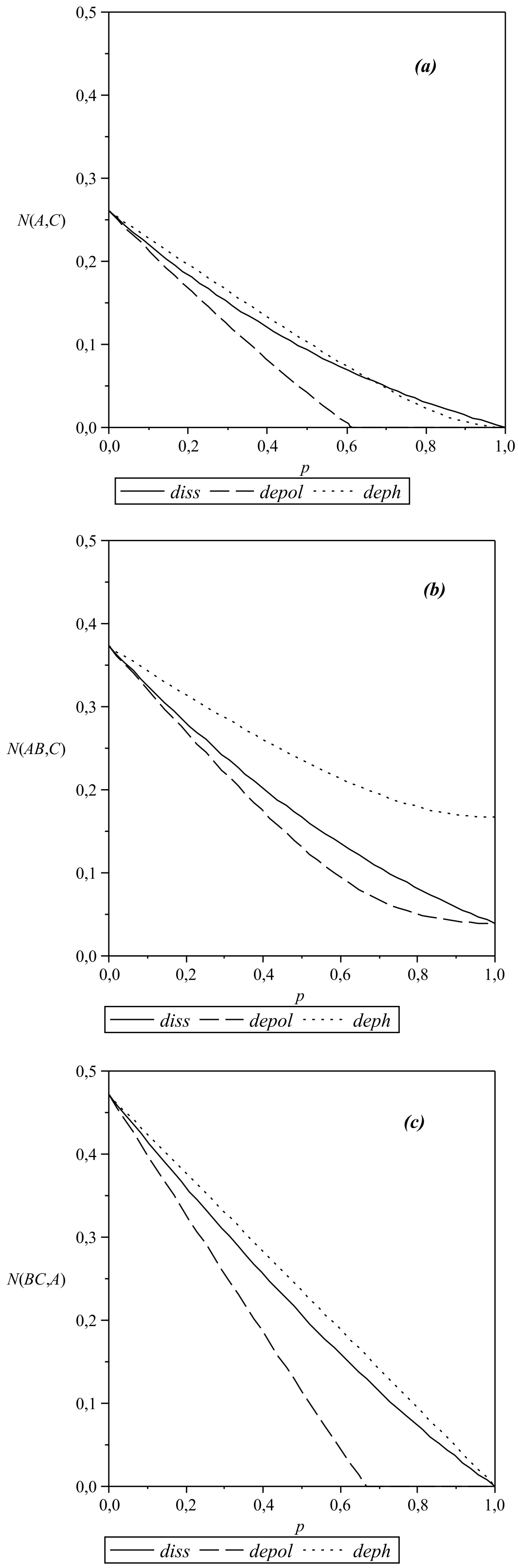

Figure 4. Negativity of the WRr state with the decohered qubit A.

We conclude that depolarization, amplifying the original causality, destroys entanglement to a lesser extent than depolarization, acting against it.

Both the conclusions coincide. Decoherence by dissipation or depolarization acting along the original causality is better from the viewpoint of entanglement persistence than acting against this causality. In other words, for entanglement persistence one should not 'stroke the system against the grain'. As a consequence, having compared the above inequalities for N and c_2, we infer that stronger entanglement corresponds to stronger causality. Of course, this inference is not universal, but it shows that less information-wise symmetric states can be more entangled.

Now consider decoherence in the partitions where a decohered qubit is in the party AC (or AB). That is, the party consists of both the original cause and effect. Thus we consider the influence of the 'internal' causality variation on entanglement in the partition AC–B in the WRr state. The corresponding curves of figures 3(c) and 4(b) evidence any of three means of decoherence at any fixed p: $N^{\mathrm{decoh}\,C} > N^{\mathrm{decoh}A}$. The inference is nontrivial: the decohered internal effect destroys entanglement to a lesser extent than the decohered internal cause.

Exactly the same inequalities proved to be valid for the decohered CKW states (and of course they are inapplicable for the originally causeless GHZ and W states). Probably, these relations between the original causal asymmetry of the quantum states and their entanglement persistence are universal.

6. Conclusion

The causal analysis formalizes an intuitive understanding of causality as an asymmetric relation and allows us to define the quantitative measure of causality. This formal definition of causality and its measure are valid in any time direction. In contrast to classical causality, the quantum one can be finite only in the mixed states, i.e. in the open systems.

We have studied the relationship between decoherence and causality. This study has been based on the consideration of the four three-qubit models decohered by dissipation, depolarization and dephasing. There are some simple regularities in the relation of decoherence with the direction and strength of causality. Dissipation always induces causality; therewith the dissipated one-qubit party tends to be the effect with respect to the rest of the system. Depolarization does not always induce causality, but if it does, the depolarized one-qubit party tends to be the cause with respect to the rest of the system. The dephasing also does not always induce causality (in a larger number of cases than depolarization) and in general acts like depolarization, but weaker.

A comparison of the measures of causality c_2 and entanglement N has shown that dissipation and depolarization acting along the original causality destroy entanglement to a lesser degree than against it. On the other hand, any kind of decoherence of the internal (inside a subsystem) effect destroys entanglement to a lesser degree than decoherence of the internal cause.

The obtained results demonstrate the relationship between entanglement decay and causal asymmetry of quantum systems. The practical use of this relationship for the protection of entangled systems against decoherence seems quite possible.

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