TIME, CLOSED TIMELIKE CURVES AND CAUSALITY

FRANCISCO LOBO[1] AND PAULO CRAWFORD[2]

Centro de Astronomia e Astrofísica da Universidade de Lisboa
Campo Grande, Ed. C8 1749-016 Lisboa, Portugal

Abstract. General Relativity is contaminated with non-trivial geometries which generate closed timelike curves. These apparently violate causality, producing time-travel paradoxes. We shall briefly discuss these geometries and analyze some of their physical aspects.

1. Introduction

It seems to be extremely difficult to give a precise definition of Time, this mysterious ingredient of the Universe. Intuitively, we have the notion of time as something that flows. Ancient religions have registered it as something unusual, and many myths are built into their dogmas.

The ancient Greeks conveyed the image of Chronos, the Father Time. Plato assumed that time had a beginning, looping back into itself. This notion of circular time, was probably inspired in the phenomena observed in Nature, namely the alternation of day and night, the repetition of the seasons, etc. But, it was in the Christian theological doctrine that the unique character of historical events gave rise to a linear notion of time. Aristotle, a keen natural philosopher, stated that time was related to motion, i.e., to change. An idea reflected in his famous metaphor: *Time is the moving image of Eternity.*

Reflections on time can be encountered in many philosophical considerations and works over the ages, culminating in Newton's Absolute Time. Newton stated that time flowed at the same rate for all observers in the Universe. But, in 1905, Einstein changed altogether our notion of time. Time flowed at different rates for different observers, and Minkowski, three years later, formally united the parameters of time and space, giving rise to the notion of a four-dimensional entity, spacetime.

Later, Einstein influenced by Mach's Principle, was motivated to seek a theory in which the structure of spacetime was influenced by the presence of matter. Therefore, according to the General Theory of Relativity, time and space would cease to exist if the Universe was empty. Adopting a pragmatic point of view, this assumption seems reasonable, as to measure time a changing configuration of matter is needed, i.e., a swinging pendulum, etc. Change seems to be imperative to have an emergent notion of time.

Therefore, time is empirically related to change. But change can be considered as a variation or sequence of occurrences. Thus, intuitively, a sequence of successive occurrences, provides us with a notion of something that flows, i.e., it provides us with the notion of *Time*. Time flows and everything relentlessly moves along this stream.

In Relativity, we can substitute the above empirical notion of a sequence of occurrences by a sequence of *events*. We idealize the concept of an event to become a point in space and an instant in time.

Following this reasoning of thought, a sequence of events has a determined *temporal order*. We experimentally verify that specific events occur before others and not vice-versa. Certain events (effects) are triggered off by others (causes), providing us with the notion of *causality*.

2. Closed timelike curves and associated paradoxes of time travel

The conceptual definition and understanding of Time, both quantitatively and qualitatively is of the utmost difficulty and importance. Special Relativity provides us with important quantitative elucidations of the fundamental processes related to time dilation effects. The General Theory of Relativity (GTR) provides a deep analysis to effects of time flow in the presence of strong and weak gravitational fields.

As time is incorporated into the proper structure of the fabric of spacetime, it is interesting to note that GTR is contaminated with non-trivial geometries which generate *closed timelike curves* [1]. A closed timelike curve (CTC) allows time travel, in the sense that an observer which travels on a trajectory in spacetime along this curve, returns to an event which coincides with the departure. The arrow of time leads forward, as measured locally by the observer, but globally he/she may return to an event in the past. This fact apparently violates causality, opening Pandora's box and producing time travel paradoxes [2], throwing a veil over our understanding of the fundamental nature of Time. The notion of causality is fundamental in the construction of physical theories, therefore time travel and it's associated paradoxes have to be treated with great caution. The paradoxes fall into two broad groups, namely the *consistency paradoxes* and the *causal loops*.

The consistency paradoxes include the classical grandfather paradox.

Imagine travelling into the past and meeting one's grandfather. Nurturing homicidal tendencies, the time traveller murders his grandfather, impeding the birth of his father, therefore making his own birth impossible. In fact, there are many versions of the grandfather paradox, limited only by one's imagination. The consistency paradoxes occur whenever possibilities of changing events in the past arise.

The paradoxes associated to causal loops are related to self-existing information or objects, trapped in spacetime. Imagine a time traveller going back to his past, handing his younger self a manual for the construction of a time machine. The younger version then constructs the time machine over the years, and eventually goes back to the past to give the manual to his younger self. The time machine exists in the future because it was constructed in the past by the younger version of the time traveller. The construction of the time machine was possible because the manual was received from the future. Both parts considered by themselves are consistent, and the paradox appears when considered as a whole. One is liable to ask, what is the origin of the manual, for it apparently surges out of nowhere. There is a manual never created, nevertheless existing in spacetime, although there are no causality violations.

3. Solutions of the EFEs generating CTCs

A great variety of solutions to the Einstein Field Equations (EFEs) containing CTCs exist, but, two particularly notorious features seem to stand out. Solutions with a tipping over of the light cones due to a rotation about a cylindrically symmetric axis; and solutions that violate the Energy Conditions of GTR, which are fundamental in the singularity theorems and theorems of classical black hole thermodynamics [3].

3.1. STATIONARY, AXISYMMETRIC SOLUTIONS

The tipping over of light cones seem to be a generic feature of some solutions with a rotating cylindrical symmetry. The general metric for a stationary, axisymmetric solution with rotation is given by [1, 4]:

$$ds^2 = -A(r)dt^2 + 2B(r)d\phi\, dt + C(r)d\phi^2 + D(r)(dr^2 + dz^2) \qquad (1)$$

in which z is the distance along the axis of rotation; r is the radial distance from the axis; ϕ is the angular coordinate; and t is the temporal coordinate. The metric components are functions of r alone. It is clear that the determinant, $g = det(g_{\mu\nu}) = -(AC + B^2)D^2$ is Lorentzian, provided that $(AC + B^2) > 0$.

Due to the periodic nature of the angular coordinate, ϕ, an azimuthal curve with $\gamma = \{t = const, r = const, z = const\}$ is a closed curve of

invariant length $s_\gamma^2 \equiv C(r)(2\pi)^2$. If $C(r)$ is negative then the integral curve with (t, r, z) fixed is a CTC.

The present work is far from making an exhaustive search of all the EFE solutions generating CTCs with these features, but the best known spacetimes will be briefly analyzed, namely, the van Stockum spacetime, the Gödel universe and spinning cosmic strings.

3.1.1. *van Stockum spacetime*

The earliest solution to the EFEs containing CTCs, is probably that of the van Stockum spacetime [1, 5]. It is a stationary, cylindrically symmetric solution describing a rapidly rotating infinite cylinder of dust, surrounded by vacuum. The centrifugal forces of the dust are balanced by the gravitational attraction. Consider R the surface of the cylinder.

The metric for the interior solution $r < R$, is given by:

$$ds^2 = -dt^2 + 2\omega r^2 d\phi\, dt + r^2(1 - \omega^2 r^2)d\phi^2 + \exp(-\omega^2 r^2)(dr^2 + dz^2) \quad (2)$$

where ω is the angular velocity of the cylinder. It is immediate to verify that CTCs arise if $\omega r > 1$. Causality violation can also be verified for $\omega R > 1/2$, in the exterior region.

3.1.2. *Spinning Cosmic String*

Consider an infinitely long straight string that lies and spins around the z-axis. The symmetries are analogous to the van Stockum spacetime, but the asymptotic behavior is different [1].

We restrict the analysis to an infinitely long straight string, with a delta-function source confined to the z-axis. It is characterized by a mass per unit length, μ; a tension, τ, and an angular momentum per unit length, J.

In cylindrical coordinates the metric takes the following form:

$$ds^2 = -\left[d(t + 4GJ\phi)\right]^2 + dr^2 + (1 - 4G\mu)^2\, r^2\, d\phi^2 + dz^2 \qquad (3)$$

Consider an azimuthal curve, i.e., an integral curve of ϕ. Closed timelike curves appear whenever $r < 4GJ/(1 - 4G\mu)$.

3.1.3. *The Gödel Universe*

Kurt Gödel in 1949 discovered an exact solution to the EFEs of a uniformly rotating universe containing dust and a nonzero cosmological constant. Writing the metric in a form in which the rotational symmetry of the solution, around the axis $r = 0$, is manifest and suppressing the irrelevant z coordinate, we have [3, 6]:

$$ds^2 = 2w^{-2}(-dt'^2 + dr^2 - (\sinh^4 r - \sinh^2 r)\, d\phi^2 + 2(\sqrt{2})\sinh^2 r\, d\phi\, dt) \quad (4)$$

Moving away from the axis, the light cones open out and tilt in the ϕ-direction. The azimuthal curves with $\gamma = \{t = const, r = const, z = const\}$ are CTCs if the condition $r > \ln(1 + \sqrt{2})$ is satisfied.

3.2. SOLUTIONS VIOLATING THE ENERGY CONDITIONS

The traditional manner of solving the EFEs, $G_{\mu\nu} = 8\pi G T_{\mu\nu}$, consists in considering a plausible stress-energy tensor, $T_{\mu\nu}$, and finding the geometrical structure, $G_{\mu\nu}$. But one can run the EFE in the reverse direction by imposing an exotic metric $g_{\mu\nu}$, and eventually finding the matter source for the respective geometry.

In this fashion, solutions violating the energy conditions have been obtained. One of the simplest energy conditions is the weak energy condition, which states: $T_{\mu\nu}U^{\mu}U^{\nu} \geq 0$, in which U^{μ} is a timelike vector. This condition is equivalent to the assumption that any timelike observer measures a local positive energy density. Although classical forms of matter obey these energy conditions, violations have been encountered in quantum field theory, the Casimir effect being a well-known example.

Adopting the reverse philosophy, solutions such as traversable wormholes, the warp drive, the Krasnikov tube and the Ori-Soen spacetime have been obtained. These solutions violate the energy conditions and with simple manipulations generate CTCs.

3.2.1. *Traversable wormholes, the Gott spacetime and the Ori-Soen solution*

Much interest has been aroused in traversable wormholes since the classical article by Morris and Thorne [7]. A wormhole is a hypothetical tunnel which connects different regions in spacetime. These solutions are multiply-connected and probably involve a topology change, which by itself is a problematic issue. One of the most fascinating aspects of wormholes is their apparent ease in generating CTCs. There are several ways to generate a time machine using multiple wormholes [1], but a manipulation of a single wormhole seems to be the simplest way [8].

An extremely elegant model of a time-machine was constructed by Gott [9]. It is an exact solution to the EFE for the general case of two moving straight cosmic strings that do not intersect. This solution produces CTCs even though they do not violate the WEC, have no singularities and event horizons, and are not topologically multiply-connected as the wormhole solution. The appearance of CTCs relies solely on the gravitational lens effect and the relativity of simultaneity.

A time-machine model was also proposed by Amos Ori and Yoav Soen which significantly ameliorates the conditions of the EFE's solutions which

generate CTCs [10]. The Ori-Soen model presents some notable features. It was verified that CTCs evolve from a well-defined initial slice, a partial Cauchy surface, which does not display causality violation. The partial Cauchy surface and spacetime are asymptotically flat, contrary to the Gott spacetime, and topologically trivial, contrary to the wormhole solutions. The causality violation region is constrained within a bounded region of space, and not in infinity as in the Gott solution. The WEC is satisfied up until and beyond a time slice $t = 1/a$, on which the CTCs appear.

3.2.2. *The Alcubierre warp drive and the Krasnikov solution*
Within the framework of general relativity, it is possible to warp spacetime in a small *bubblelike* region [11], in such a way that the bubble may attain arbitrarily large velocities, $v(t)$. Inspired in the inflationary phase of the early Universe, the enormous speed of separation arises from the expansion of spacetime itself. The model for hyperfast travel is to create a local distortion of spacetime, producing an expansion behind the bubble, and an opposite contraction ahead of it.

One may consider a spaceship immersed within the bubble, moving along a timelike curve, regardless of the value of $v(t)$. Due to the arbitrary value of the warp bubble velocity, the metric of the warp drive permits superluminal travel. This possibility raises the question of the existence of CTCs. Although the solution deduced by Alcubierre by itself does not possess CTCs, Everett demonstrated that these are created by a simple modification of the Alcubierre metric [12], by applying a similar analysis as in tachyons.

Krasnikov discovered an interesting feature of the warp drive, in which an observer in the center of the bubble is causally separated from the front edge of the bubble. Therefore he/she cannot control the Alcubierre bubble on demand. Krasnikov proposed a two-dimensional metric [13], which was later extended to a four-dimensional model [14]. Using two such tubes it is a simple matter, in principle, to generate CTCs.

4. Conclusion

GTR has been an extremely successful theory, with a well established experimental footing, at least for weak gravitational fields. It's predictions range from the existence of black holes, gravitational radiation to the cosmological models, predicting a primordial beginning, namely the big-bang.

However, it was seen that it is possible to find solutions to the EFEs, with certain ease, which generate CTCs. This implies that if we consider GTR valid, we need to include the *possibility* of time travel in the form of CTCs. A typical reaction is to exclude time travel due to the associated

paradoxes. But the paradoxes do not prove that time travel is mathematically or physically impossible. Consistent mathematical solutions to the EFEs have been found, based on plausible physical processes. What they do seem to indicate is that local information in spacetimes containing CTCs are restricted in unfamiliar ways.

The grandfather paradox, without doubt, does indicate some strange aspects of spacetimes that contain CTCs. It is logically inconsistent that the time traveller murders his grandfather. But, one can ask, what exactly impeded him from accomplishing his murderous act if he had ample opportunities and the free-will to do so. It seems that certain conditions in local events are to be fulfilled, for the solution to be globally self-consistent. These conditions are denominated *consistency constraints* [15]. To eliminate the problem of free-will, mechanical systems were developed as not to convey the associated philosophical speculations on free-will [16]. Much has been written on two possible remedies to the paradoxes, namely the Principle of Self-Consistency and the Chronology Protection Conjecture.

One current of thought, led by Igor Novikov, is the Principle of Self-Consistency, which stipulates that events on a CTC are self-consistent, i.e., events influence one another along the curve in a cyclic and self-consistent way. In the presence of CTCs the distinction between past and future events are ambiguous, and the definitions considered in the causal structure of well-behaved spacetimes break down. What is important to note is that events in the future can influence, but cannot change, events in the past.

The Principle of Self-Consistency permits one to construct local solutions of the laws of physics, only if these can be prolonged to a unique global solution, defined throughout non-singular regions of spacetime. Therefore, according to this principle, the only solutions of the laws of physics that are allowed locally, reinforced by the consistency constraints, are those which are globally self-consistent.

Hawking's Chronology Protection Conjecture is a more conservative way of dealing with the paradoxes. Hawking notes the strong experimental evidence in favour of the conjecture from the fact that "we have not been invaded by hordes of tourists from the future" [17].

An analysis reveals that the value of the renormalized expectation quantum stress-energy tensor diverges in the imminence of the formation of CTCs. This conjecture permits the existence of traversable wormoles, but prohibits the appearance of CTCs. The transformation of a wormhole into a time machine results in enormous effects of the vacuum polarization, which destroys it's internal structure before attaining the Planck scale. There is no convincing demonstration of the Chronology Protection Conjecture, but the hope exists that a future theory of quantum gravity may prohibit CTCs.

Visser still considers the possibility of two other conjectures [1]. The first

is the radical reformulation of physics conjecture, in which one abandons the causal structure of the laws of physics and allows, without restriction, time travel, reformulating physics from the ground up. The second is the boring physics conjecture, in which one simply ceases to consider the solutions to the EFEs generating CTCs.

Perhaps an eventual quantum gravity theory will provide us with the answers. But, as stated by Thorne [18], it is by extending the theory to it's extreme predictions that one can get important insights to it's limitations, and probably ways to overcome them. Therefore, time travel in the form of CTCs, is more than a justification for theoretical speculation, it is a conceptual tool and an epistemological instrument to probe the deepest levels of GTR and extract clarifying views.

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