WHY IS QUANTUM MECHANICS NON-LOCAL?

A question on a correspondence between the famous Bell's Theorem and Quantum Mechanics foundations is formulated. The statement is argued that the Quantum Mechanics non-locality is due to its predicted nonlinear dependence on an argument, for instance – on the angle difference between two analyser orientations.

Introduction

The most of people doesn't think about some philosophic problems while is going to do something. Similarly, the most of physicists who is solving a practical problems of Quantum Mechanics (QM) doesn't interesting in discussions on its conceptual foundations. However, sooner or later a moment comes when these abstract questions gain suddenly in importance.

A perception that QM – and namely as a theoretical discipline – in principle violates the traditional relativistic requirements of local causality came several decades after its creation. The basic contribution was made here by Albert Einstein with his co-authors **[EPR, 1935]** and by John Bell **[Bell, 1964]**. Bell analyzed EPR-paper and also tried to solve the QM *incompleteness* problem, but like Columbus (who found America instead of India) he discovered the QM *non-locality*. So, he stated his famous inequality that can be violated in QM.

There are some other theoretical facts confirming the QM non-locality. Before all I talk about the Feynmann formalism based on path integrals. As it is well known, a particle moves (if its path isn't detected) not along one determined trajectory, but by the all possible ways. In the Feynmann QM formalism a possible way with a maximal statistical weigt is corresponding to a classical path that is predicted by a variational principle of mechanics.

I would like to present one more important exemple of the QM non-locality. As it is known, any QM non-zero commutator includes the universal constant ħ (Planck's constant). In my paper **[Shulman, 2004]** I showed that for purely classical oscillators one may deduce an analogous commutation expressions. Instead of the constant ħ at right hand they contain the action function, i.e. the product of the coordinate and momentum amplitudes for this concrete oscillator. I can give the only one explanation of this fact – every quantum oscillator is non-local and extends over the all Universe. Therefore, the constant ħ seems to be proportional to the current Universe (finite) perimeter.

Let us consider now a QM prediction for a polarised photon probability to transverse two series analyzers (polarisers) having the angle difference θ. As it is known, this probability is proportional to $\cos^2\theta$, and *it isn't depending on the distance between analyzers*. May it be that this relationship is approximate, and it is true only for short distances? Is it possible that for a long distances the light velocity (or another one) should be accounted? No, this statement is exact as theoretical one, and it is surely tested in the famous experiments **[Aspect, 2000]**. So, we come to the standard sentence: QM predictions violate the Bell's Inequality, then QM is incompatible with Local Reality.

But this statement seems to be unexpected, because the Bell's Theorem at first view is not related with any QM background. It is typical, I don't know some papers discussing the *causes* of this situations.

At least, two questions appear:

(1) Are in principle possible another theories and predictions conflicting with Local Reality too?

(2) What are namely basic QM statements that imply with necessity its non-locality?

I will try to answer below these questions after brief describing of the Bell's results and Aspect's experiments, it is needed as methodical background.

Bell's Theorem and Aspect's experiments

Let us consider the Bell's ideas with more details. I will directly follow the very tacit paper **[Aspect, 2000]**. Bell introduced some *local hidden variable* (or set of such variables) that he denoted as λ. Further, let us consider two EPR photons which are flying from one source to the opposite sides. At one side the first photon is to be detected by analyzer I, and at the second side another photon is to be detected by analyzer II. Bell defined a probability distribution of λ over the pair ensemble by function $\rho(\lambda)$. A result for any pair attributing by a value λ is determined by two quantities A and B that can have only two possible polarisation values: +1 and –1

$$A(\lambda,\mathbf{a}) = \pm 1 \quad \text{at analyzer I (with orientation } \mathbf{a}\text{)}$$

$$B(\lambda,\mathbf{b}) = \pm 1 \quad \text{at analyzer II (with orientation } \mathbf{b}\text{)}$$

A concrete local hidden variable theory is completely determined by an explicit form of functions $\rho(\lambda)$, $A(\lambda,\mathbf{a})$, and $B(\lambda,\mathbf{b})$. Using these ones we can express a different measurement result probabilities. For example, the correlation function is defined as

$$E(\mathbf{a}, \mathbf{b}) = \int d\lambda\, \rho(\lambda)\, A(\lambda,\mathbf{a})\, B(\lambda,\mathbf{b})$$

Now let us consider an important quantity

$$s = A(\lambda,\mathbf{a})\, B(\lambda,\mathbf{b}) - A(\lambda,\mathbf{a})\, B(\lambda,\mathbf{b'}) + A(\lambda,\mathbf{a'})\, B(\lambda,\mathbf{b}) + A(\lambda,\mathbf{a'})\, B(\lambda,\mathbf{b'}) =$$
$$= A(\lambda,\mathbf{a})\, [B(\lambda,\mathbf{b}) - B(\lambda,\mathbf{b'})] + A(\lambda,\mathbf{a'})\, [B(\lambda,\mathbf{b}) + B(\lambda,\mathbf{b'})]$$

Because A and B can only be equal to ±1, then

$$s(\lambda, \mathbf{a}, \mathbf{a'}, \mathbf{b}, \mathbf{b'}) = \pm 2$$

The averaging s over λ gives us that this quantity must only have a value between +2 and –2:

$$-2 \leq \int d\lambda\, \rho(\lambda)\, s(\lambda, \mathbf{a}, \mathbf{a'}, \mathbf{b}, \mathbf{b'}) \leq 2$$

Now we can present these inequalities in the form

$$-2 \le S(\mathbf{a}, \mathbf{a'}, \mathbf{b}, \mathbf{b'}) \le 2$$

where

$$S(\mathbf{a}, \mathbf{a'}, \mathbf{b}, \mathbf{b'}) = E(\mathbf{a}, \mathbf{b}) - E(\mathbf{a}, \mathbf{b'}) + E(\mathbf{a'}, \mathbf{b}) + E(\mathbf{a'}, \mathbf{b'})$$

Aspect says:

> These are famous Bell's inequalities generalized by Clauser, Horne, Shimony, Holt. They bear upon the combination S of the four polarisation correlation coefficients, associated to two directions of analysis for each polariser (**a** and **a'** for polariser I, **b** and **b'** for polariser II)… The most important hypothesis, stressed by Bell in all his papers, is the local character of the formalism. We have indeed implicitly assumed that the result $A(\lambda, \mathbf{a})$ of the measurement at polariser I does not depend on the orientation **b** of the remote polariser II, and vice-versa. Similarly, it is assumed that the probability distribution $\rho(\lambda)$ (i.e. the way in which pairs are emitted) does not depend on the orientations **a** and **b**. This *locality assumption* is crucial: Bell's Inequalities would no longer hold without it.

However, QM predicts the expression:

$$S_{QM}(\mathbf{a}, \mathbf{a'}, \mathbf{b}, \mathbf{b'}) = \cos 2(\mathbf{a},\mathbf{b}) - \cos 2(\mathbf{a},\mathbf{b'}) + \cos 2(\mathbf{a'},\mathbf{b}) + \cos 2(\mathbf{a'}, \mathbf{b'})$$

It depends on three independent variables only (**a**,**b**), (**b**,**a'**) и (**a'**,**b'**). In fact, the fourth one may be expressed through three remaining ones:

$$(\mathbf{a},\mathbf{b'}) = (\mathbf{a},\mathbf{b}) + (\mathbf{b},\mathbf{a'}) + (\mathbf{a'}, \mathbf{b'})$$

It is obviously from symmetry reasons that extremums of functions S will reach at equal values of angles (**a,b**), (**b,a'**), and (**a', b'**). Then we can denote each of them by a value θ, and search for extremums of the univariate function only:

$$S_{QM}(\theta) = 3\cos 2\theta - \cos 6\theta$$

We will reach extremums at condition

$$\sin\theta = \sin 3\theta$$

The plot **[Aspect, 2000]** of this univariate function $S_{QM}(\theta)$ is showed on the fig. 1. A conflict with the Bell's inequalities appears at $|S_{QM}| > 2$.

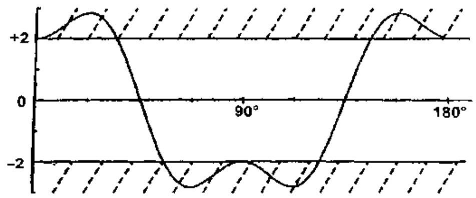

Figure 1

The author of the Ref. **[Aspect, 2000]** with his co-authors in the Optic Institute (Paris) performed a long serie of experiments during several years, and surely confirmed the non-local features of QM. The Bell's Inequalities synchronised test was performed by switching each polariser orientation at random moments.

The total testing consisted in 8000 s of data accumulation with 4 polarisers … The final result was reached

$$S_{exp} = 0.101 \pm 0.020$$

violating the upper limit of the Bell's inequality by 5 standard deviations, and in good agreement with the Quantum Mechanics predictions:

$$S_{QM} = 0.113 \pm 0.005$$

Quantum Mechanics Non-locality Origin

In order to answer the first question of two that are formulated in the Introduction, let us consider the very important examle from Ref. **[Aspect, 2000]**. It consists in a local classical model with a pair of photons, each of them moves along the axis Oz and has a well determined linear polarisation that is defined by an angle λ with axis O*x*. The both photons have the same angle value λ due to their common origin, so the correlation is strong. The polarisation of the various pairs is randomly distributed, and we take this distribution rotationally invariant. Further, let θ_I and θ_{II} indicate the orientations of the polarisers, $A(\lambda,\mathbf{a})$ assumes the value +1 when the polarisation of photon 1 makes an angle less than π/4 with the direction of analysis **a**, and −1 for the complementary case (polarisation closer to the perpendicular to **a**). Similarly for photon 2 and $B(\lambda,\mathbf{b})$. One can calculate a different measurement result probabilities and the correlation function.

The calculation remarkable result is showed below on the fig. 2. The difference beween the predictions of the simple classical model and the quantum mechanical predictions is always small, and the agreement is exact for the angles 0, ±π/4 и ±π/2 (a strong correlation). However, this difference is very important.

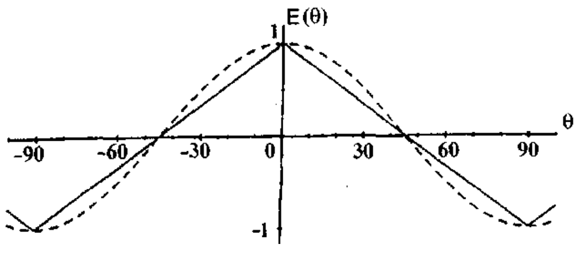

Figure 2

Polarisation correlation coefficient E(θ), as a function of the relative orientation of the polarisers **[Aspect, 2000]**: dotted line – Quantum Mechanical prediction; solid line – the classical local model.

For such classical local model the correlation coefficient is giving by the relationship

$$E(\mathbf{a},\mathbf{b}) = 1 - 4|(\mathbf{a},\mathbf{b})| / \pi \ , \qquad \text{where} \quad -\pi/2 \le (\mathbf{a},\mathbf{b}) \le \pi/2$$

So, the univariate function $S_{loc}(\theta)$ is everywhere indepenent on the angle θ and doesn't violate the Bell's ineqyalities:

$$S_{loc}(\theta) = 3E(\theta) - E(\mathbf{3}\theta) = 3(1 - 4|\theta|/\pi) - (1 - 12|\theta|/\pi) = 2$$

Note: We used 4=3+1 different angles for function $S(\theta)$, one of them is the sum of the *three* remaining ones. Behaps, one can take any value N > 1 instead of 3. Then we will have $S_{loc}(\theta) = NE(\theta) - E(N\theta)$, it will be always equal to $S(\theta) = N - 1$ for the classical model.

It looks as if independence $S(\theta)$ on the angle θ is mamely the crucial factor for the local model that determines in principle the difference between theories with hidden variables and other ones like QM. However, the $S(\theta)$ independence on θ is directly due to linearity $S(\theta_1 + \theta_2) = S(\theta_1) + S(\theta_2)$. If such, then namely *nonlinear* probability dependence on the argument in QM leads its non-locality.

Lets us again consider the Aspect's example. There are the angles θ_I and θ_{II} indicating polariser directions. In fact, the locality condition requires to use some other arguments – the differencies $(\theta_I - \theta)$ and $(\theta_{II} - \theta)$, where θ is a common inititial condition. Further, we are interesting in the coincidence function (or the correlation coefficient) $F\{(\theta_I - \theta), (\theta_{II} - \theta)\}$, and we hope that this function will finally have the form $F\{\theta_I - \theta_{II}\}$, i.e. the common initial value θ will "disappear" from final relationship similarly to the QM prediction. But for any values θ_I and θ_{II} it is always possible only if this function is linear

$$F\{(\theta_I - \theta), (\theta_{II} - \theta)\} = A[(\theta_I - \theta) - (\theta_{II} - \theta)] + B = A(\theta_I - \theta_{II}) + B$$

that is exatly accomplished in the case of the Aspect's example.

Because QM predicts the nonlinear function for the coincidence probability in the form $\cos^2(\theta_I - \theta_{II})$, then it can't be a theory with a common local type condition. Clearly, *any* theory predicting nonlinear dependence on the difference $(\theta_I - \theta_{II})$ can't be a such type theory too.

The next question we posed – why do QM give namely nonlinear prediction relative to $(\theta_I - \theta_{II})$ – is obviously linked with the QM axiomatics, state pointers and angles between them (in real space and in Hilbert's one).

Bibliography

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